District of Columbia

Mathematics

Pre-K through Grade 12 Standards





PROPOSED | *March 7, 2005*

District of Columbia
Mathematics
Pre-K – Grade 12 Standards

PROPOSED

March 7, 2005

INTRODUCTION

The following Mathematics Standards are the work of many District of Columbia teachers and administrators, early childhood providers, community stakeholders, parents, and others in a process that was coordinated by the office of the Chief Academic Officer, working with StandardsWork, Inc. They were adapted from standards used in the Commonwealth of Massachusetts and a set of Pre-K standards developed for the District by a team working under the direction of the Executive Director of the Office of Early Childhood Development.

The learning standards specify what students should know and be able to do as learners of mathematics at the end of each grade level or course. Students are held responsible for learning standards listed at earlier grades as well as their current grade.

Organization of the document:

This document is organized into three parts:

- **A statement of guiding philosophies that articulate a set of beliefs about teaching, learning, and assessing mathematics in District of Columbia;**
- **Grade-by-grade standards, organized as described below; and**
- **A glossary that explains words and phrases found in the standards.**

The mathematics learning standards for pre-kindergarten through grade eight are organized by grade level and presented in five strands:

1. **Number Sense and Operations (pp. 6–15);**
2. **Patterns, Relations, and Algebra (pp. 16–19);**
3. **Geometry (pp. 20–23);**
4. **Measurement (pp. 24–27); and**
5. **Data Analysis, Statistics and Probability (pp. 28–31**).

The standards for grades nine through twelve are organized differently. The mathematics studied in high school fall naturally under the discipline headings:

- **Algebra I (pp. 33–34);**
- **Geometry (pp. 35–36);**
- **Algebra II (pp. 37–38);**
- **Probability and Statistics (p. 39); and**
- **Precalculus (pp. 40–41).**

To allow schools and teachers flexibility, the standards do not mandate that a particular high school course be initiated and completed in a single grade. For example, students could take Geometry in grade nine, ten, or eleven, depending on the preferred sequence of course offerings at each high school.

Each learning standard in every grade or course has a unique identifier that consists of:

- Grade level: Pre-K, K, 1, 2, 3, etc. or course AI = Algebra I, G = Geometry, AII = Algebra II, PS= Probability and Statistics, and PC = Precalculus;
- Strand: N = Number Sense and Operations, P = Patterns, Relations, and Algebra, G = Geometry, M = Measurement, and D = Data Analysis, Statistics, and Probability; and
- Standard number.

For example, standard 4.G.6 is the sixth standard of the Geometry strand in grade 4. Standard AI.N.12 is the twelfth standard of the Number Sense and Operations strand in the Algebra I course. This numbering system allows teachers to also organize the standards by grade. For example, fifth grade teachers preparing their curriculum can distinguish grades 5 standards in each strand by identifying all of the standards beginning with a 5.

Although the District of Columbia presents standards for only two post-Algebra II courses—Probability and Statistics and Precalculus—there are other mathematics courses that schools might offer concurrent with or subsequent to Precalculus. Among these options are discrete mathematics and calculus. Schools should also provide interested students with enrichment options in mathematics such as advanced placement courses, independent research, internships, or opportunities to study special topics.

Guiding Philosophies for the Teaching and Learning of Mathematics[1]

The following grade-specific standards envision all students in the District of Columbia achieving mathematical competence through a strong mathematics program that emphasizes problem solving, communicating, reasoning and proof, making connections, and using representations. These skills must be woven throughout the five strands of mathematics: Number Sense and Operations; Patterns, Relationships, and Algebra; Geometry; Measurement; and Data Analysis and Probability.

An effective mathematics program focuses on problem solving and requires teachers who have a deep knowledge of mathematics as a discipline.

Problem solving is both a means of developing students' knowledge of mathematics and a critical outcome of a good mathematics education. As such, it is an essential component of the curriculum. A mathematical problem, as distinct from an exercise, requires the solver to search for a method for solving the problem rather than following a set procedure. Mathematical problem solving, therefore, requires an understanding of relevant concepts, procedures, and strategies. To become able problem solvers, students need many opportunities to formulate questions, model problem situations in a variety of ways, generalize mathematical relationships, and solve problems in both mathematical and everyday contexts. They also must have a firm grasp of mathematical techniques and their underlying principles. Students need to be able to distinguish relevant from irrelevant information, identify missing information, and either find what is needed or make appropriate estimates. When solving problems, students need to be able to test ideas, try different approaches, explain their reasoning, check their results for errors and reasonableness of solutions and devise independent ways to verify results. Mathematical problem solving calls for reflective thinking, persistence, learning from the ideas of others, and going back over one's own work with a critical eye. Success in solving mathematical problems helps to create an abiding interest in mathematics.

Effective mathematics programs depend on knowledgeable teachers. To promote achievement of these standards, teachers need to encourage classroom talk, reflection, use of multiple problem solving strategies, and a positive disposition toward mathematics. They also need to hold high expectations for all students. Teachers need to design to challenge students in multiple ways. Short- and long-term investigations that connect procedures and skills with conceptual understanding are integral components of an effective mathematics program.

The study of mathematics is an exercise in reasoning that must go beyond acquiring procedural mathematical skills with their clear methods and boundaries. Students need to master the more subjective skills of reading, interpreting, representing, and communicating a problem and its solution.

From the early grades on, students develop their reasoning skills by making and testing mathematical conjectures, drawing logical conclusions, and justifying their thinking in developmentally appropriate ways. In the early grades, for example, repeated addition becomes multiplication, multiplication of numbers less than ten can be extended to numbers less than one hundred and then to the entire number system, and knowing how to find the area of a right triangle extends to all right triangles. As they advance through the grades, students' arguments become more sophisticated, and they are able to use inductive and deductive reasoning to arrive at valid conclusions and construct simple proofs. By doing so, students learn what mathematical reasoning entails.

The ability to express mathematical ideas coherently to different audiences is an important skill in a technological society. Students develop this skill and deepen their understanding of mathematics when they use accurate mathematical language to talk and write about what they are doing. They clarify mathematical ideas as they discuss them with peers and reflect on strategies and solutions. By talking and writing about mathematics and using the special symbols of mathematics correctly and precisely, students learn how to make convincing arguments and to represent mathematical ideas verbally, pictorially, and symbolically. When they can do so, they have a set of tools that significantly expands their capacity to think mathematically.

Mathematics is not a collection of separate strands or standards. Rather, it is an integrated field of study. Students develop a perspective of the mathematics field as an integrated whole by understanding connections within and outside of the discipline. This includes the ability to shift regularly between the specific and general (i.e., use specific examples to understand general ideas; extend specific results to more general cases). Major emphasis should be given to ideas and concepts across mathematical content areas that help students see that mathematics is a web of closely connected ideas. It is also important for teachers to demonstrate the significance and relevance of the subject by encouraging students to explore the connections that exist within mathematics, with other disciplines, and between mathematics and students' own experiences.

Technology is an essential tool in a mathematics education.

---

[1] Adapted from the Massachusetts Mathematics Curriculum Framework, November 2000

Technology enhances the mathematics curriculum in many ways. Tools such as measuring instruments, manipulatives (such as base ten blocks and fraction pieces), scientific and graphing calculators, and computers with appropriate software, if properly used, contribute to a rich learning environment for developing and applying mathematical concepts. However, *appropriate use of calculators is essential*; calculators should not be used as a replacement for basic understanding and skills. Elementary students should learn how to perform thoroughly the basic arithmetic operations independent of the use of a calculator. Although the use of a graphing calculator can help middle and secondary students to visualize properties of functions and their graphs, graphing calculators should be used to enhance their understanding and skills rather than replace them.

Technology enables students to communicate ideas within the classroom and to search for information in external data bases such as the Internet, an important supplement to a school's internal library resources. Technology can be especially helpful in assisting students with special needs in regular and special classrooms, at home, and in the community. Technology changes what mathematics is to be learned and when and how it is learned. For example, currently available technology provides a dynamic approach to such mathematical concepts as functions, rates of change, geometry, and averages that was not possible in the past. Some mathematics becomes more important because technology requires it; some becomes less important because technology replaces it; and some becomes possible because technology allows it.

Strand 1: Number Sense and Operations

Pre-Kindergarten
PK.N.1. Use one-to-one correspondence (e.g., sees four children at table and gives each child one cup. Touches each doll as she counts how many are in the cradle). PK.N.2. Count with understanding to at least 10 (e.g., counts 10 blocks, pointing to each as he counts and then says, "I have ten!" Chooses and counts 7 beads to put on necklace). PK.N.3. Use numbers to tell how many (number quantity) (e.g., says, "I broke my cookie into four pieces." Takes attendance and says, "There are ten boys and nine girls"). PK.N.4. Use numbers and counting as a means to solve problems, predict, and measure quantities (e.g., says, "Five cups" when asked to predict how many cups it will take to fill the bucket. Says, "Only four kids can ride tricycles now because that's all there are"). PK.N.5. Recognize and name numerals up to 10 (e.g., points to each number on the toy clock while counting aloud. Points to sign and says, "See, only four kids can be at the water table"). PK.N.6. Quickly recognize quantity of small groups of objects up to 4 (e.g., sees 3 bear counters and says, "There are three of them," without having to count them. While getting ready to paint at the easel, says, "Why are there only three paint colors today? We always have four!"). PK.N.7. Construct sets of a given number using concrete objects (e.g., counts six blocks to match the numeral 6. Plays a game of dominoes with a friend, lining up sides with the same number of dots to each other). PK.N.8. Demonstrate the idea of adding and subtracting by using concrete objects (e.g., while playing "Bears in a Cave," says, "I see two bears, so one must be hiding." Arranges 3 teddy bear counters in a block construction and then gets 1 more, saying, "Now I have 4"). PK.N.9. Use ordinal numbers and positional words in everyday activities (e.g., looks at picture schedule and describes what comes first, second, and third. Arranges objects in order (seriate) from small to large).

Strand 1: Number Sense and Operations

Kindergarten	Grade One	Grade Two
Number Sense K.N.1. Count by ones to at least 20. K.N.2. Represent, name, and order a set of objects (up to 20). K.N.3. Match quantities up to at least 10 with numerals and words. K.N.4. Compare sets of up to at least 10 concrete objects using appropriate language (e.g., none, more than, fewer than, same number of, one more than). K.N.5. Identify positions of objects in sequences (e.g., first, second) up to fifth. K.N.6. Identify US coins by name and determine their value. Fractions K.N.7. Understand the concepts of whole and half. Computation and Operations K.N.8. Use objects and drawings to model and solve related addition and subtraction problems to 10. Estimation K.N.9. Estimate the number of objects in a group and verify results.	Number Sense 1.N.1. Count, read, and write whole numbers to 110 and relate them to the quantities they represent (e.g., knows that 60 is bigger than 20). 1.N.2. Compare and order whole numbers to 110 by using symbols for less than, equal to, or greater than (<, =, >). 1.N.3. Identify the place value of the digits to 110. 1.N.4. Represent equivalent forms of the same number through the use of physical model, diagrams, and number expressions (e.g., 9 may be represented as 4+5, 3+6, 3+3+3, 10-1, 12-3). 1.N.5. Use concrete objects (manipulatives) to model odd and even numbers and determine whether a set of objects has an odd or even number of elements. 1.N.6. Make combinations of different coins up to 50 cents. Fractions 1.N.7. Model, identify, and represent fractions such as 1/2, 1/3, 1/4 as parts of wholes (e.g., one-fourth of a pie), parts of groups, and numbers on the number line.	Number Sense 2.N.1. Count, read, and write whole numbers to 1,000 and relate them to the quantities they represent. 2.N.2. Compare and order numbers to 1000; use the symbols >, <, =. 2.N.3. Identify the place value of the digits to 1000. 2.N.4. Use words, models, and expanded forms (e.g., 35 = 3 tens + 5 ones) to represent numbers to 1000. 2.N.5. Know that even numbers end in 0, 2, 4, 6,or 8; recognize even numbers as multiples of 2; know that odd numbers end in 1, 3, 5, 7 or 9 and work with patterns involving even and odd numbers. 2.N.6. Identify the value of all US coins and $1, $5, $10 and $20 bills. Find the value of a collection of coins and dollar bills and different ways to represent an amount of money up to $5. Fractions 2.N.7. Know that fractions may represent a portion of a whole that has been partitioned into parts of equal area or length; use the terms "numerator" and "denominator." 2.N.8. Recognize the inverse relationship between the size of a unit fraction and the size of the denominator (i.e., the larger the denominator, the smaller the size of the unit fraction). 2.N.9. Recognize, name, and write commonly used fractions such as 1/2, 2/3, 3/4. 2.N.10. Recognize that fractions such as 2/2, 3/3, 4/4, 10/10, 100/100 are equal to the whole and to one.

Strand 1: Number Sense and Operations, cont'd.

Kindergarten	Grade One	Grade Two
	<u>Computation and Operations</u> 1.N.8. Demonstrate the ability to use conventional algorithms for addition and subtraction. 1.N.9. Demonstrate an understanding of various meanings of addition and subtraction, such as addition as combination (i.e., plus, combined with, more); subtraction as comparison (i.e., how much less, how much more), equalizing (i.e., how many more are needed to make these equal), and separation (i.e., how much remaining). 1.N.10. Know addition and subtraction facts (addends to ten), commit to memory, and use them to solve problems. 1.N.11. Demonstrate the ability to add and subtract one- and two-digit numbers fluently without regrouping. 1.N.12. Use mental arithmetic to find the sum or difference of two 1-digit numbers. 1.N.13. Find the sum of three 1-digit numbers (e.g., 3 + 4 + 2 =). 1.N.14. Identify one more than, one less than, 10 more than, and 10 less than for any number up to 100. 1.N.15. Understand and use the inverse relationship between addition and subtraction (e.g., 8 + 6 = 14 is equivalent to 14 – 6 = 8 and is also equivalent to 14 – 8 = 6) to solve problems and check solutions. <u>Estimation</u> 1.N.16. Recognize when an estimate is reasonable in problems that involve numbers that use the ones, tens, hundreds, and thousands places.	<u>Computation and Operations</u> 2.N.11. Demonstrate the ability to use conventional algorithms for addition (two 3-digit numbers and three 2-digit numbers) and subtraction (two 3-digit numbers). 2.N.12. Find the distance between numbers on the number line (e.g., how far is 76 from 24). 2.N.13. Know addition and subtraction facts (addends to twelve), commit to memory, and use them to solve problems. 2.N.14. Demonstrate the ability to add and subtract 3-digit numbers accurately and efficiently. 2.N.15. Use mental arithmetic to find the sum or difference of two 2-digit numbers. 2.N.16. Represent multiplication as repeated addition 2.N.17. Demonstrate proficiency with multiplication facts for the 1's, 2's and 5's. 2.N.18. Demonstrate an understanding of the inverse relationship of addition and subtraction and use that understanding to simplify computation and check solutions. <u>Estimation</u> 2.N.19. Estimate, calculate, and solve problems involving addition and subtraction of 2-digit numbers. Describe differences between estimates and actual calculations.

Strand 1: Number Sense and Operations

Grade Three	Grade Four	Grade Five
Number Sense 3.N.1. Exhibit an understanding of the base ten number system by reading, modeling, and writing whole numbers to at least 10,000; demonstrate an understanding of the values of the digits. 3.N.2. Represent, compare, and order numbers to 10,000 using various forms, including expanded notation (e.g., 3,206= 3 x 1,000 + 2 x 100 +6) and written out in words (e.g., three thousand two-hundred six). 3.N.3. Round whole numbers through 10,000 to the nearest 10, 100, and 1,000. 3.N.4. Recognize sets to which a number may belong (odd numbers, even numbers, and multiples of numbers through 10). Identify the numbers in those classes (e.g., the class of multiples of 7 between 1 and 29 consists of 7, 14, 21, 28).	Number Sense 4.N.1. Exhibit an understanding of the base ten number system by reading, modeling, and writing whole numbers to at least 100,000, demonstrating an understanding of the values of the digits, and comparing and ordering the numbers. 4.N.2. Represent, compare, and order numbers to 100,000 using various forms, including expanded notation. 4.N.3. Round whole numbers to 100,000 to the nearest 10, 100, 1,000, 10,000, and 100,000. 4.N.4. Recognize sets to which a number may belong (odds, evens, multiples and factors of given numbers, and squares) and use these in the solution of problems. 4.N.5. Read and interpret whole numbers and decimals up to two decimal places; relate to money and place-value decomposition. 4.N.6. Determine if a whole number is a multiple of a given 1-digit whole number and if a 1-digit number is a factor of a given whole number. 4.N.7. Find all factors of a whole number up to 50; know that numbers such as 2, 3, 5, 7, and 11 do not have any factors except 1 and itself and that such numbers are called prime numbers.	Number Sense 5.N.1. Estimate, round, and manipulate very large (e.g., billions) and very small (e.g., thousandths) numbers; demonstrate an understanding of place value to billions and thousandths. 5.N.2. Represent and compare very large (billions) and very small (thousandths) positive numbers in various forms such as expanded notation without exponents (e.g., 9724 = 9 x 1000 + 7 x 100 + 2 x 10 + 4). 5.N.3. Find and position integers, fractions, mixed numbers, and decimals (both positive and negative) on the number line. 5.N.4. Compare and order integers (including negative integers) and positive fractions, mixed numbers, decimals, and percents. 5.N.5. Apply the number theory concepts of common factor, common multiple, and divisibility rules for 2, 3, 5, and 10 to the solution of problems. Demonstrate an understanding of the concepts of prime and composite numbers. 5.N.6. Know the set of prime numbers to 100. 5.N.7. Determine the prime factors of all numbers through 50 and write the numbers as the product of their prime factors by using exponents to show multiples of a factor (e.g., $24=2x2x2x3=2^3x3$).

Strand 1: Number Sense and Operations, continued

Grade Three	Grade Four	Grade Five
Fractions and Decimals 3.N.5. Identify and represent fractions (between 0 and 1 with denominators through 10) as parts of unit wholes and parts of a collection. 3.N.6. Recognize, name, and use equivalent fractions with denominators 2, 3, 4, and 8; place these fractions on the number line; compare and order them and relate the number line to a ruler, e.g., 1/2=2/4=4/8). 3.N.7. Know the meaning of 0.75, 0.50, and 0.25 as they relate to money; know that fractions and decimals are two different representations of the same concept (e.g., 50 cents is 1/2 of a dollar, 75 cents is 3/4 of a dollar). 3.N.8. Know that any fraction can be written as a sum of unit fractions (e.g., 3/4=1/4+1/4+1/4). 3.N.9. Model and represent a mixed number (with denominator 2, 3, or 4) as a whole number and a fraction (e.g., 1 2/3, 3 ½).	Fractions and Decimals 4.N.8. Demonstrate an understanding of fractions as parts of unit wholes, as parts of a collection, as locations on a number line, and as locations on the number line. 4.N.9. Know the relationships among halves, fourths, and eighths and among thirds, sixths, and twelfths; compare and order such fractions. 4.N.10. Recognize, name, and generate equivalent forms of common decimals (0.5, 0.25, 0.2, 0.1) and fractions (halves, quarters, fifths, and tenths) and explain why they are equivalent. 4.N.11. Select, use and explain models to relate common fractions and mixed numbers (e.g., 1/2, 1/3, 1/4, 1/5, 1/8, 1/10, 1/12, and 1 1/2); find equivalent fractions, mixed numbers, and decimals. 4.N.12. Represent decimals between 0 and 1 up to the hundredths.	Fractions, Decimals, and Percents 5.N.8. Explain different interpretations of fractions as a ratio of whole numbers, as parts of unit wholes, as parts of a collection, as division of whole numbers by whole numbers, as locations on the number line. 5.N.9. Interpret percents as parts out of 100, use % notation, and express a part of a whole as a percentage. 5.N.10. Identify and determine common equivalent fractions, mixed numbers (with denominators 2, 4, 5, 10), decimals, and percents and explain why they represent the same value. 5.N.11. Write improper fractions as mixed numbers, and know that a mixed number represents the number of "wholes" and the part of a whole remaining (e.g., 5/4 = 1+1/4=1 1/4).

Strand 1: Number Sense and Operations, cont'd.

Grade Three	Grade Four	Grade Five
Computation and Operations 3.N.10. Demonstrate an understanding of and the ability to use conventional algorithms for the addition and subtraction of up to 5-digit numbers. 3.N.11. Add and subtract up to 4-digit numbers accurately and efficiently. 3.N.12. Use concrete objects and visual models to add and subtract common fractions (halves, thirds, fourths, sixths, and eighths) with like denominators. 3.N.13. Solve problems involving addition and subtraction of money amounts in decimal notation. 3.N.14. Know multiplication is the result of counting the total number of objects in a set of equal groups (e.g., 3 x 5 gives the number of objects in 3 groups of 5 objects). 3.N.15. Know division (÷) as another way of expressing multiplication, i.e., that division "undoes" multiplication (e.g., 2 x 3 = 6 can be rewritten as 6 ÷ 2 = 3 or 6 ÷ 3 = 2). 3.N.16. Know multiplication facts through 10 x 10 and related division facts (e.g., 9 x 8 = 72 and 72 ÷ 9 = 8). Use these facts to solve related problems (e.g., 3 x 5 is related to 3 x 50) 3.N.17. Solve simple problems involving multiplication of multidigit numbers by one-digit numbers (2,431 x 2). 3.N.18. Solve division problems in which a multidigit number is evenly divided by a one-digit number (e.g., 125 ÷ 5). 3.N.19. Multiply up to 2-digit numbers by a 1-digit number accurately and efficiently.	Computation and Operations 4.N.13. Demonstrate an understanding of and the ability to use conventional algorithms for the addition and subtraction of multidigit numbers. 4.N.14. Add and subtract up to 5-digit numbers accurately and efficiently. 4.N.15. Use concrete objects and visual models to add and subtract fractions where the denominators are equal or when one denominator is a multiple of the other (denominators 2 through 12, and 100). 4.N.16. Select, use, and explain various meanings and models of multiplication and division of whole numbers. Understand and use the inverse relationship between the two operations. 4.N.17. Know multiplication facts through 12 x 12 and the inverse division facts. Use these facts to solve related multiplication problems and compute related problems. 4.N.18. Demonstrate understanding of and ability to use the conventional algorithms for multiplication of up to three digits by two digits. Multiply 3-digit numbers by two digits accurately and efficiently. 4.N.19. Demonstrate understanding of and the ability to use the conventional algorithm for division of up to three digits with a single-digit divisor (with or without remainders). Divide up to a 3-digit number with a single-digit divisor accurately and efficiently. Interpret any remainders. 4.N.20. Multiply fractions by whole numbers, using repeated addition and area rectangular models.	Computation and Operations 5.N.12. Add with negative integers, subtract positive integers from negative integers, and verify the reasonableness of the results. 5.N.13. Add and subtract fractions (including mixed numbers) with like and unlike denominators (of 2,3,4, 10 only) and express answers in the simplest form. 5.N.14. Add and subtract positive decimals. 5.N.15. Solve problems involving multiplication and division of any whole number. 5.N.16. Demonstrate proficiency with division, including division with positive decimals and long division with multidigit divisors. 5.N.17. Use models to show an understanding of multiplication and division of fractions; multiply positive fractions with whole numbers. Simplify fractions in cases when both the numerator and the denominator have 2, 3, 4, 5, or 10 as a common factor. 5.N.18. Multiply positive decimals with whole numbers. 5.N.19. Demonstrate an understanding of and compute (positive integer) powers of ten (e.g., 10^2, 10^5); compute examples as repeated multiplication.

Strand 1: Number Sense and Operations, cont'd

Grade Three	Grade Four	Grade Five
3.N.20. Use the commutative (order) and identity properties of addition and multiplication on whole numbers in computations and problem situations (e.g., 3 + 4 + 7 = 3 + 7 + 4 = 10 + 4). 3.N.21. Know and apply the special properties of 0 and 1 in multiplication. 3.N.22. Use multiplication and division fact families to understand the inverse relationship of these two operations and to compare and check results (e.g., because 3 x 8 = 24, we know that 24 ÷ 8 = 3 or 24 ÷ 3 = 8). Estimation 3.N.23. Estimate the sum and difference of two numbers with three digits (sums up to 1000) and judge reasonableness of estimates. 3.N.24. Understand and use the strategies of rounding and regrouping to estimate quantities, measures, and the results of whole-number computations (addition, subtraction, and multiplication) up to 2-digit whole numbers and amounts of money to $100, and to judge the reasonableness of answers.	4.N.21. Mentally calculate simple products and quotients up to a 3-digit number by a 1-digit number (e.g., 400 x 7, or 320 ÷ 8). 4.N.22. Multiply and divide money amounts in decimal notation by using whole-number multipliers and divisors. 4.N.23. Determine the unit cost when given the total cost and number of units. 4.N.24. Select and use appropriate operations (addition, subtraction, multiplication, and division) to solve problems, including those involving money. 4.N.25. Select, use, and explain the commutative, associative, and identity properties of operations on whole numbers in problem situations, e.g., 37 x 46 = 46 x 37, (5 x 7) x 2 = 5 x (7 x 2). 4.N.26. Use the relationship between multiplication and division to simplify computations and check results. Estimation 4.N.27. Estimate and compute the sum or difference of whole numbers and positive decimals to two places. 4.N.28. Estimate the answers to calculations involving addition, subtraction, or multiplication; know when approximation or a rounded solution is appropriate and use it to check the reasonableness of answers. 4.N.29. Select and use a variety of strategies (e.g., front-end, rounding, and regrouping) to estimate quantities, measures, and the results of whole-number computations up to 3-digit whole numbers and amounts of money to $1000, and to judge the reasonableness of answers.	5.N.20. Demonstrate an understanding of how parentheses affect expressions involving addition, subtraction, and multiplication, and use that understanding to solve problems, e.g., 3 x (4 + 2) = 3 x 6. Estimation 5.N.21. Estimate sums and differences of whole numbers, positive fractions, and positive decimals. Estimate products of whole numbers and products of positive decimals with whole numbers. Use a variety of strategies and judge reasonableness of answers.

Strand 1: Number Sense and Operations

Grade Six	Grade Seven	Grade Eight
Number Sense 6.N.1. Explain the properties of and compute with rational numbers, expressed in a variety of forms. 6.N.2. Compare and order positive and negative fractions, decimals, and mixed numbers and place them on a number line. 6.N.3. Know that numbers and their negatives add to 0 and are on opposite sides and at equal distance from 0 on a number line; know that 0 is an integer that is neither negative nor positive. 6.N.4. Represent rational numbers as fractions or terminating decimals when possible and translate between these representations. 6.N.5. Identify and determine common equivalent fractions, mixed numbers, decimals, and percents. 6.N.6. Apply number theory concepts–including prime and composite numbers, prime factorization, greatest common factor, least common multiple, and divisibility rules for 2, 3, 4, 5, 6, 9, and 10–to the solution of problems. 6.N.7. Round whole numbers and decimals to any given place.	Number Sense 7.N.1. Compare, order, estimate, and translate among integers, fractions, and mixed numbers (i.e., rational numbers), decimals, and percents. 7.N.2. Know that in decimal form, rational numbers either terminate or eventually repeat and that calculators truncate or round repeating decimals; locate rational numbers on the number line; convert between common repeating decimals and fractions. 7.N.3. Demonstrate an understanding of absolute value, e.g., $\|-3\| = \|3\| = 3$. 7.N.4. Represent numbers in scientific notation (positive powers of ten only) and use that notation in problem situations. 7.N.5. Differentiate between rational and irrational numbers (i.e., those that cannot be expressed as the quotient of two integers and cannot be represented by terminating or repeating decimals). 7.N.6. Interpret positive whole-number powers as repeated multiplication and negative powers as repeated division or multiplication by the multiplicative inverse. Simplify and evaluate expressions that include exponents. 7.N.7. Apply number theory concepts, including prime factorization and relatively prime numbers, to the solution of problems (e.g., find the prime factorization of whole numbers and write the results using exponents: $24 = 2 \times 2 \times 2 \times 3 = 2^3 \times 3$). 7.N.8. Express ratios in several ways (e.g., 3 cups to 5 people; 3:5; 3/5); recognize and find equivalent ratios. 7.N.9. Know the meaning of a square root of a number and its connection to the square whose area is the number.	Number Sense 8.N.1. Explain the properties of and compute with real numbers, expressed in a variety of forms. 8.N.2. Know that every rational number is either a terminating or repeating decimal and that every irrational number is a nonrepeating decimal. 8.N.3. Understand that computations with an irrational number and a rational number (other than zero) produce an irrational number. 8.N.4. Know that the absolute value is the distance of the number from 0; determine the absolute value and additive inverse of real numbers; determine the absolute value of rational numbers. 8.N.5. Read, write, and compare rational numbers in scientific notation (positive and negative powers of 10) and use them in calculations and problem situations. 8.N.6. Define, compare, order, and apply frequently used irrational numbers, such as $\sqrt{2}$ and π. 8.N.7. Use the laws of exponents for integer exponents (e.g., write $2^2 \times 2^3$ as $2 \times 2 \times \ldots$ and then as a single power of 2; write 2^{-3} as a fraction). 8.N.8. Identify and use the arithmetic properties of subsets of integers and rational numbers.

Strand 1: Number Sense and Operations, cont'd.

Grade Six	Grade Seven	Grade Eight
Computation and Operations 6.N.8. Select and use appropriate operations to solve problems involving addition, subtraction, multiplication, division, and positive integer exponents with whole numbers and with positive fractions, mixed numbers, decimals, and percents. 6.N.9. Know integer subtraction is the inverse of integer addition; use the number line to model addition and subtraction of integers and add and subtract integers, with the exception of subtracting negative integers. 6.N.10. Accurately and efficiently add, subtract, multiply, and divide (with multidigit divisors) whole numbers and positive decimals. 6.N.11. Use prime factorization to add and subtract fractions with like and unlike denominators. 6.N.12. Accurately and efficiently add, subtract, multiply, and divide positive fractions (including mixed numbers) with like and unlike denominators. Simplify fractions. 6.N.13. Calculate given percentages of quantities and solve problems involving discounts at sales, interest earned, and tips. 6.N.14. Apply laws of exponents to multiply whole number powers with like bases.	Computation and Operations 7.N.10. Estimate and compute with fractions (including simplification of fractions), integers, decimals, and percents (including those greater than 100 and less than 1) using the four operations and combinations of the four operations. 7.N.11. Demonstrate an understanding of the properties of arithmetic operations on rational numbers (integers, fractions, and terminating decimals); convert terminating decimals into reduced fractions. 7.N.12. Select and use appropriate operations—addition, subtraction, multiplication, division, and positive integer exponents—to solve problems with rational numbers and negative integers. 7.N.13. Calculate the percentage increase and decrease of a quantity. 7.N.14. Use ratios and proportions in the solution of problems involving unit rates, scale drawings, and reading of maps. . 7.N.15. Take positive and negative rational numbers to positive whole number powers. 7.N.16. Apply the laws of exponents to multiply whole number positive and negative powers of whole numbers; divide whole number powers with like bases; explain the inverse relationship between negative and positive exponents.	Computation and Operations 8.N.9. Calculate weighted averages such as course grades, consumer price indexes, and sports ratings. 8.N.10. Solve problems involving ratio units such as miles per hour, dollars per pound, or persons per square mile. 8.N.11. Solve problems involving derived quantities such as density, velocity, and weighted averages. 8.N.12. Solve simple proportion problems using such methods as unit rate, scaling, finding equivalent fractions, and solving the proportion equation $a/b = c/d$. 8.N.13. Solve problems that involve markups, commissions, profits, and simple and compound interest. 8.N.14. Apply the rules of powers and roots to the solution of problems. 8.N.15. Use the inverse relationship between squaring and finding the square root of a perfect square integer to solve problems. 8.N.16. Multiply and divide numbers written in scientific notation.

Strand 1: Number Sense and Operations, cont'd.

Grade Six	Grade Seven	Grade Eight
6.N.15. Apply the Order of Operations for expressions involving addition, subtraction, multiplication, and division with grouping symbols (+, –, x, ÷). Estimation 6.N.16. Estimate results of computations with whole numbers and with positive fractions, mixed numbers, decimals, and percents. Describe reasonableness of estimates.	7.N.17. Use the inverse relationships of addition and subtraction, multiplication and division to simplify computations and solve problems (e.g., multiplying by 1/2 or 0.5 is the same as dividing by 2). 7.N.18. Use the associative, commutative, and distributive properties; properties of the identity and inverse elements (e.g., -7 + 7 = 0; 3/4 x 4/3 = 1). 7.N.19. Extend the Order of Operations to include positive integer exponents. Estimation 7.N.20. Estimate results of computations with rational numbers; determine when an estimate rather than an exact answer is appropriate and apply in problem situations.	8.N.17. Select and use appropriate operations—addition, subtraction, multiplication, division, and positive integer exponents—to solve problems with rational numbers, including negative rationals. Estimation 8.N.18. Estimate and solve problems with square roots; find square roots of perfect squares and approximate the square roots of non-perfect squares by locating them between consecutive integers. 8.N.19. Determine when an estimate rather than an exact answer is appropriate and apply in problem situations.

Strand 2: Patterns, Relations, and Algebra

Pre-Kindergarten
PK.P.1. Sort and classify objects by more than one attribute – color, shape, size, number, etc. (e.g., sorts play dough cookies by size, color, or shape. Sorts a collection of buttons into those with 1- 4 holes). PK.P.2. Recognize, describe, and copy simple patterns (e.g., joins the teacher in a clapping pattern, *slap the knees, slap the knees, clap hands; slap the knees, slap the knees, clap hands.* Uses a stamp to repeat a pattern).

Strand 2: Patterns, Relations, and Algebra

Kindergarten	Grade One	Grade Two
K.P.1. Identify the attributes of objects as a foundation for sorting and classifying (e.g., a red truck, a red block, and a red ball share the attribute of being red; a square block, a square cracker, and a square book share the attribute of being square). K.P.2. Sort and classify objects by attribute such as color, shape, size, number, and other properties, and explain; identify objects that do not belong to a particular group (e.g., all these objects are red; those are green). K.P.3. Identify, reproduce, describe, extend, and create color, rhythmic, shape, number, and letter repeating patterns with simple attributes. K.P.4. Count by fives and tens up to at least 50.	1.P.1. Identify, reproduce, describe, extend, and create simple rhythmic, shape, size, number, color, and letter repeating patterns. I.P.2. Describe and create addition and subtraction number patterns (e.g., 1, 4, 7, 10. . . or 25, 23, 21. . .). I.P.3. Identify different patterns on the hundreds chart. 1.P.4. Skip count forward and backward by twos, fives, and tens up to at least 50, starting at any number and using appropriate aids (e.g., hundreds chart, number line). 1.P.5. Write and solve number sentences from problem situations that express relationships involving addition and subtraction, including +, -, <, >, =. 1.P.6. Apply knowledge of fact families to solve simple open sentences for addition and subtraction that have variables (e.g., □ + 2 = 7 and 10 - □ =6).	2.P.1. Distinguish between repeating and growing patterns; create and describe patterns such as repeating patterns and growing patterns using number, shape, size, color, and letter. 2.P.2. Describe functions related to coin trades and measurement trades (e.g., five pennies make one nickel; four cups make one quart). 2.P.3. Skip count forward and backward by twos, fives, and tens up to at least 100, starting at any number. 2.P.4. Construct and solve open sentences that have variables (e.g., 42 + □ = 57). 2.P.5. Use the commutative and associative rules for addition to simplify mental calculations and to check results.

Strand 2: Patterns, Relations, and Algebra

Grade Three	Grade Four	Grade Five
3.P.1. Create, describe, extend, and explain symbolic (geometric) patterns and addition and subtraction patterns; describe patterns in a variety of ways. 3.P.2. Use boxes or other symbols to represent unknowns or quantities that vary in expressions and in equations or inequalities (=, < and >). 3.P.3. Select appropriate operational and relational symbols to make an expression true (e.g., if 4_3=12, what operational symbol goes in the blank?). 3.P.4. Determine values of variables in simple equations involving addition, subtraction, or multiplication (e.g., 4106 – ∇ = 37, 5 = μ + 3, and □ – μ = 3). 3.P.5. Know and express the relationships among linear units of measure, i.e., unit conversions (e.g., 3 feet=1 yard; 12 inches=1 foot). 3.P.6. Extend and recognize a linear pattern by its rules (e.g., the number of legs on a given number of horses may be calculated by counting by 4s or by multiplying the number of horses by 4).	4.P.1. Create, describe, extend, and explain geometric and numeric patterns, including multiplication patterns like 3, 30, 300, 3000; generalize the rule for the pattern and make predictions when given a table of number pairs of a set of data. 4.P.2. Use symbol and letter variables (e.g., Δ, x) to represent unknowns or quantities that vary in expressions and in equations or inequalities (mathematical sentences that use =, < and >). 4.P.3. Use pictures, models, tables, charts, graphs, words, number sentences, and mathematical notations to interpret mathematical relationships. 4.P.4. Solve problems involving proportional relationships, including unit pricing (e.g., four apples cost 80 cents, so one apple costs 20 cents) and map interpretation (e.g., one inch represents five miles, so two inches represent ten miles). 4.P.5. Determine how change in one variable relates to a change in a second variable (e.g., input-output tables).	5.P.1. Analyze and determine the rules for extending symbolic, arithmetic, and geometric patterns and progressions (e.g., ABBCCC…; 1, 5, 9, 13, …; 3, 9, 27, …). 5.P.2. Replace variables with given values, evaluate and simplify (e.g., 2(μ) + 3 when μ = 4). 5.P.3. Use the properties of equality to solve problems with whole numbers (e.g., if x + 7 = 13, then x = 13 – 7, therefore x = 6; if 3 x □ = 15, then $^1/_3$ x 3 x □ = $^1/_3$ x 15, therefore □ = 5). 5.P.4. Represent real situations and mathematical relationships with concrete models, tables, graphs, and rules in words and with symbols (e.g., input-output tables). 5.P.5. Interpret and evaluate mathematical expressions that use parentheses; use parentheses to indicate which operation to perform first when writing expressions containing more than two terms and different operations. 5.P.6. Solve problems involving proportional relationships using concrete models, tables, graphs, and paper-pencil methods. 5.P.7. Interpret graphs that represent the relationship between two variables in everyday situations.

Strand 2: Patterns, Relations, and Algebra

Grade Six	Grade Seven	Grade Eight
6.P.1. Use the properties of equality to solve problems using letter name variables (e.g., 1/4 + x = 7/12). 6.P.2. Solve linear equations using concrete models, tables, graphs, and paper-pencil methods. 6.P.3. Identify and describe relationships between two variables with a constant rate of change (e.g., perimeter-side relationship for a square, distance-time graphs, and conversions such as feet to inches). Contrast these with relationships where the rate of change is not constant. 6.P.4. Simplify expressions of the first degree by combining like terms, and evaluate using specific values. 6.P.5. Understand that adding or subtracting the same number to both sides of an equation creates a new equation that has the same truth values. 6.P.6. Understand that multiplying or dividing both sides of an equation by the same non-zero number creates a new equation that has the same truth values. 6.P.7. Distinguish between an algebraic expression and an equation. 6.P.8. Recognize when information given in a table, graph, or formula suggests a proportional or linear relationship. 6.P.9. Produce and interpret graphs that represent the relationship between two variables (x and y) in everyday situations.	7.P.1. Extend, represent, analyze, and generalize a variety of patterns with tables, graphs, words, and when possible, symbolic expressions. Include arithmetic and geometric progressions (e.g., compounding). 7.P.2. Evaluate simple algebraic expressions for given variable values (e.g., $3a^2 - b$ for a = 3 and b = 7). 7.P.3. Use the correct order of operations to evaluate expressions (e.g., 3(2x) = 5). 7.P.4. Create and use symbolic expressions for linear relationship and relate them to verbal, tabular, and graphical representations. 7.P.5. Use variables and appropriate operations to write an expression, an equation, or an inequality that represents a verbal description (e.g., three less than a number, half as large as area A). 7.P.6. Solve linear equations using tables, graphs, models, and algebraic methods. 7.P.7. Identify, describe, and analyze linear relationships between two variables. Compare positive rate of change (e.g., $y = 3x + 1$) to negative rate of change (e.g., $y = -3x + 1$). 7.P.8. Use linear equations to model and analyze problems involving proportional relationships. Use technology as appropriate. 7.P.9. Simplify numerical expressions by applying properties of rational numbers (e.g., identity, inverse) and operations of rational numbers (distributive, associative, commutative); justify the process used. 7.P.10. Use algebraic terminology including but not limited to variable, equation, term, coefficient, inequality, expression, constant. 7.P.11. Plot the values of quantities whose ratios are always the same (e.g., cost to the number of an item, feet to inches, circumference to diameter of a circle). Fit a line to the plot and understand that the slope of the line equals the quantities.	8.P.1. Use tables and graphs to represent and compare linear growth patterns. In particular, compare rates of change and x- and y-intercepts of different linear patterns. 8.P.2. Set up and solve linear equations and inequalities with one or two variables using algebraic methods, models, and/or graphs. 8.P.3. Use linear equations to model and analyze problems involving proportional relationships. Use technology as appropriate. 8.P.4. Identify the slope of a line as a measure of its steepness and as a constant rate of change from its table of values, equation, or graph. Apply the concept of slope to the solution of problems. 8.P.5. Identify the roles of variables within an equation (e.g., y = mx + b, expressing y as a function of x with parameters m and b). 8.P.6. Distinguish between numerical and algebraic expressions, equations, and inequalities. 8.P.7. Demonstrate an understanding of the identity (-x)(-y) = xy. Use this identity to simplify algebraic expressions (e.g., (-2)(-x+2) = 2x – 4). 8.P.8. Explain and analyze—both quantitatively and qualitatively, using pictures, graphs, charts, and equations—how a change in one variable results in a change in another variable in functional relationships (e.g., $C = \pi d$, $A = \pi r^2$ (A as a function of r), $A_{rectangle} = lw$ ($A_{rectangle}$ as a function of l and w). 8.P.9. Graph a linear equation using ordered pairs; identify and represent the graphs of linear functions.

Strand 3: Geometry

Pre-Kindergarten
PK.G.1. Describe, name, and interpret distance and position in space; understand and use positional words (e.g., turns Lotto game board so player sitting opposite can see it right side up. Frustrated, says, "I can't reach the ball; it's too high").
PK.G.2. Recognize, name, and describe simple two- and three-dimensional shapes (e.g., says, "this is a triangle. See, it has three sides." Says, "You need balls of clay to make a snowman").
PK.G.3. Match, sort, and classify shapes (e.g., says, "these all go together because they have three sides." When cleaning up blocks, orders the different shapes on the shelf by matching them to the outlines on the shelf).
PK.G.4. Put together and take apart shapes to make new shapes (e.g., makes a picture using a variety of pattern block shapes. Puts a straw across a square and says, "Now I have triangles").
PK.G.5. Create shapes using concrete materials, such as straws (e.g., uses toothpicks to make rectangles of different sizes. Puts a ball on top of a triangular block and says, "I'm eating an ice cream cone").

Strand 3: Geometry

Kindergarten	Grade One	Grade Two
K.G.1. Name shapes of pattern blocks (e.g., triangle, square, hexagon, rhombus, trapezoid) K.G.2. Describe attributes of two-dimensional shapes (e.g., number of sides, number of corners, size, roundness); sort these shapes. K.G.3. Identify and compare three-dimensional shapes (e.g., cone, cube, cylinder, sphere). K.G.4. Identify positions of objects in space and use appropriate language (e.g., beside, inside, next to, close to, above, below, apart) to describe and compare their relative positions.	1.G.1. Describe attributes and parts of two- and three-dimensional shapes (e.g., length of sides and number of corners, edges, faces, and sides). 1.G.2. Identify congruent shapes. 1.G.3. Identify symmetry in two-dimensional shapes. 1.G.4. Combine shapes and take them apart to make other shapes (e.g., two congruent right triangles can be arranged to form a rectangle). 1.G.5. Arrange and describe objects in space by proximity, position, and direction (e.g., near, far, below, above, up, down, behind, in front of, next to, left or right of).	2.G.1. Identify, describe, draw, and compare two-dimensional shapes, including both polygonal (up to six sides) and curved figures such as circles. 2.G.2. Classify familiar two- and three-dimensional shapes by common attributes such as shape of curved and straight lines, number and shape of faces, edges, and vertices. 2.G.3. Match and construct congruent (e.g., two triangles that are the same shape and size) and symmetric shapes (e.g., two halves of a heart divided down the center line). 2.G.4. Identify shapes under rotation (turns), reflections (flips), translation (slides), and enlargement. Describe direction of translations (e.g., left, right, up, down). 2.G.5. Predict and explain the results of putting two-dimensional shapes together and taking them apart (e.g., two congruent right triangular shapes form a rectangle). 2.G.6. Relate geometric ideas to numbers (e.g., seeing rows in an array as a model of repeated addition).

Strand 3: Geometry

Grade Three	Grade Four	Grade Five
3.G.1. Compare and analyze attributes and other features (e.g., number and shape of sides, faces, corners, right angles) of two-dimensional geometric shapes, especially the attributes of triangles (isosceles, equilateral, right) and quadrilaterals (rectangle, square, parallelogram). 3.G.2. Describe, model, draw, compare, and classify two-dimensional shapes such as circles and polygons, especially triangles and quadrilaterals. 3.G.3. Identify angles as right, acute (less than a right angle), or obtuse (greater than a right angle). 3.G.4. Identify and draw parallel, perpendicular, and intersecting lines. 3.G.5. Identify and draw lines of symmetry in two-dimensional shapes. 3.G.6. Apply techniques such as reflections (flips), rotations (turns), and translations (slides) for determining if two shapes are congruent. 3.G.7. Using ordered pairs of whole numbers and/or letters, locate and identify points on a grid.	4.G.1. Compare and analyze attributes and other features (e.g., number of sides, faces, corners, right angles, diagonals, and symmetry) of two- and three-dimensional geometric shapes. 4.G.2. Describe, model, draw, compare, and classify two- and three-dimensional shapes (e.g., circles, polygons, cubes, spheres, pyramids, cones, cylinders). 4.G.3. Know the definitions of a right angle, an acute angle, and an obtuse angle. Understand that 90°, 180°, 270°, and 360° are associated, respective with 1/4, 1/2, 3/4, and full turns. 4.G.4. Describe and draw intersecting, parallel, and perpendicular lines. 4.G.5. Recognize similar figures (two shapes, R and S, are similar if they are congruent after one of them is shrunk or expanded). 4.G.6. Describe and apply techniques such as reflections (flips), rotations (turns), and translations (slides) for determining if two shapes are congruent. 4.G.7. Predict and validate the results of partitioning, folding, and combining two- and three-dimensional shapes. 4.G.8. Using ordered pairs of numbers and/or letters, graph, locate, and identify points and describe paths (first quadrant).	5.G.1. Identify polygons based on their properties, including types of interior angles, perpendicular or parallel sides, and congruence of sides (e.g., squares, rectangles, rhombuses, parallelograms, trapezoids, and isosceles, equilateral, and right triangles). 5.G.2. Identify, describe, and compare special types of three-dimensional shapes (e.g., cubes, prisms, spheres, cones, and pyramids) based on their properties, such as edges and faces. 5.G.3. Identify relationships among points, lines, and planes (e.g., intersecting, parallel, perpendicular). 5.G.4. Identify and describe types of symmetry, including line and rotational. 5.G.5. Determine if two triangles or two quadrilaterals are congruent by measuring sides or a combination of sides and angles. 5.G.6. Predict, describe, and perform transformations on two-dimensional shapes (e.g., translations, rotations, and reflections). 5.G.7. Graph points and identify coordinates of points on the Cartesian coordinate plane in the first two quadrants.

Strand 3: Geometry

Grade Six	Grade Seven	Grade Eight
6.G.1. Match three-dimensional objects and their two-dimensional representations (e.g., nets, projections, and perspective drawings). 6.G.2. Identify angles as vertical, adjacent, complementary, or supplementary; provide descriptions of these terms; and use the properties of complementary and supplementary angles and the sum of the angles of a triangle to solve problems involving an unknown angle. 6.G.3. Determine if two shapes are congruent by motions or series of motions (e.g., translations, rotations, and reflections); predict the results of transformations on unmarked planes and draw the transformed figure (e.g., predict how tessellations transform under translation, reflection, and rotation). 6.G.4. Graph points and identify coordinates of points on the Cartesian coordinate plane in all four quadrants. 6.G.5. Find the distance between two points on horizontal or vertical number lines.	7.G.1. Identify three-dimensional figures (e.g., prisms, pyramids) by their physical appearance, distinguishing attributes, and spatial relationships such as parallel faces. 7.G.2. Demonstrate an understanding of conditions that indicate two geometrical figures are congruent and what congruence means about the relationships between the sides and angles of the two figures. 7.G.3. Classify figures in terms of congruence and similarity, and apply these relationships to the solution of problems. 7.G.4. Use compass, straightedge, and protractor to perform basic geometric constructions to draw polygons and circles. 7.G.5. Understand and use coordinate graphs to plot simple figures, determine lengths and areas related to them, determine their image under translations, reflections, and rotations (e.g., predict how tessellations transform under translations, reflections, and rotations).	8.G.1. Analyze, apply, and explain the relationship between the number of sides and the sums of the interior and exterior angle measures of polygons. 8.G.2. Demonstrate an understanding of the relationships of angles formed by intersecting lines, including parallel lines cut by a transversal. 8.G.3. Demonstrate an understanding of conditions that indicate two triangles are similar: the corresponding angles are congruent (AAA similarity); the ratios of two pairs of corresponding sides are equal and the included angles are congruent (SAS similarity); ratios of all pairs of corresponding sides are equal (SSS similarity). 8.G.4. Know and understand the Pythagorean theorem and its converse. Apply the theorem to the solution of problems, including perimeter, area, and volume problems. 8.G.5. Use a straightedge, compass, protractor, or other tools to formulate and test conjectures and to draw geometric figures (e.g., the perpendicular bisector of a segment, an equilateral triangle, the bisector of an angle, diagonals, midpoints, radii, diameters, and chords of circles). 8.G.6. Apply spatial reasoning by recognizing and drawing two-dimensional representations of three-dimensional objects (e.g., nets, projections, and perspective drawings of cylinders, prisms, cones). 8.G.7. Find the distance between two points on the coordinate plane using the distance formula; find the midpoint of the line segment; recognize that the distance formula is an application of the Pythagorean theorem.

Strand 4: Measurement

Pre-Kindergarten
PK.M.1. Identify appropriate tools of measurement (e.g., picks up a measuring cup and says, "We need to add two cups of water to the cake mix," in dramatic play. Experiments using a balance scale to see how many wooden cubes make one side go all the way down). PK.M.2. Make use of nonstandard and standard units for measurement to obtain information (e.g., uses footsteps to measure the length of the hopscotch grid on the playground. Looks at the clock and asks, "Is it time to go outside?"). PK.M.3. Show awareness of time concepts and sequence (e.g., says, "After lunch we have read aloud time." Says, "We go home at 3 o'clock").

Strand 4: Measurement

Kindergarten	Grade One	Grade Two
K.M.1. Recognize and compare objects with respect to the attributes of length, volume/capacity, weight, area, and time using appropriate language (e.g., longer, taller, shorter, same length; heavier, lighter, same weight; holds more, holds less, holds the same amount). K.M.2. Make and use estimates of measurements from everyday experiences. K.M.3. Use standard and nonstandard units to measure length, area, weight, and capacity. K.M.4. Order events in a day. K.M.5. Tell time to the nearest hour. K.M.6. Identify US coins and their value.	1.M.1. Compare the length, weight, and volume of two or more objects by using direct comparison. 1.M.2. Make and use estimates of measurement, including time and weight. 1.M.3. Measure the length of objects by repeating a nonstandard or standard unit. 1.M.4. Tell time at half-hour intervals on analog and digital clocks using a.m. and p.m. and relate time to events (e.g., before/after, shorter/longer). 1.M.5. Make combinations of coins up to 50 cents.	2.M.1. Measure and compare the length of common objects using metric and US Customary units to the nearest centimeter or inch. 2.M.2. Make and use estimates of measurement including time, volume, weight, and area. 2.M.3. Select and correctly use the appropriate measurement tool (ruler, balance scale, thermometer). 2.M.4. Tell time at quarter-hour intervals. 2.M.5. Identify parts of the day (e.g., morning, afternoon, evening), days of the week, and months of the year. Identify dates using a calendar. 2.M.6. Identify the value of all US coins and $1, $5, $10, and $20 bills. Find the value of a collection of coins and bills and different ways to represent an amount of money up to $5 using appropriate notation.

Strand 4: Measurement

Grade Three	Grade Four	Grade Five
3.M.1. Demonstrate an understanding of such attributes as length, area, and weight; select the appropriate type of unit for measuring each attribute using both the US Customary and metric systems. 3.M.2. Carry out simple unit conversions within a system of measurement such as hours to minutes and cents to dollars (e.g., one hour = 60 minutes). 3.M.3. Identify time to the nearest five minutes on analog and digital clocks using a.m. and p.m. Compute elapsed time using a clock (e.g., hours and minutes since…) and using a calendar (e.g., days since…). 3.M.4. Estimate and find area and perimeter of a rectangle and triangle using diagrams, models, and grids or by measuring.	4.M.1. Identify and use appropriate metric and US Customary units and tools (e.g., ruler, protractor, graduated cylinder, thermometer) to estimate, measure, and solve problems involving length, area, volume, weight, time, angle size, and temperature. 4.M.2. Carry out simple unit conversions within a system of measurement (e.g., yards to feet or inches; gallons to quarts and pints). 4.M.3. Identify time to the minute on analog and digital clocks using a.m. and p.m. Compute elapsed time using a clock (e.g., hours and minutes since…) and using a calendar (e.g., days since…). 4.M.4. Estimate and find area and perimeter of shapes, including irregular shapes, using diagrams, models, and grids or by measuring. 4.M.5. Recognize that rectangles that have the same area can have different perimeters; understand that rectangles that have the same perimeter can have different areas.	5.M.1. Apply the concepts of perimeter and area to the solution of problems involving triangles and rectangles. Apply formulas where appropriate. 5.M.2. Apply formulas for the areas of triangles, rectangles, and parallelograms; recognize that shapes with the same number of sides but different appearances can have the same area. 5.M.3. Solve problems involving proportional relationships and units of measurement. 5.M.4. Identify, measure, and describe circles and the relationships of the radius, diameter, circumference, and area (e.g., d=2r) and use these concepts to solve problems. 5.M.5. Find volumes and surface areas of rectangular prisms. 5.M.6. Know that angles on a straight line add up to 180°, interior angles of a triangle add up to 180°, angles surrounding a point add up to 360°, and interior angles of a quadrilateral add up to 360°; use these properties to solve problems. 5.M.7. Identify, measure, describe, classify, and draw various angles and triangles, given sides and the angle between them or given two angles and the side between them (e.g., draw a triangle with one right angle and two sides congruent).

Strand 4: Measurement

Grade Six	Grade Seven	Grade Eight
6.M.1. Differentiate between and use appropriate units of measures for two- and three-dimensional objects (i.e., when finding perimeter, area, and volume). 6.M.2. Find areas of triangles and parallelograms. Recognize that shapes with the same number of sides but different appearances can have the same area. 6.M.3. Develop strategies to find the area and perimeter of complex shapes (e.g., subdividing them into basic shapes such as quadrilaterals, triangles, circles). 6.M.4. Solve problems involving proportional relationships and units of measurement (e.g., same system unit conversions, scale models, maps, and speed). 6.M.5. Understand the concept of volume; use the appropriate units in common measuring systems (e.g., cubic inch, cubic centimeter, cubic meter, cubic yard) to compute the volume of rectangular solids, including rectangular prisms. 6.M.6. Identify, measure, describe, classify, and construct various angles, triangles, and quadrilaterals; measure the interior angles of various polygons. 6.M.7. Understand the concept of a constant such as π; know the formulas for the circumference and area of a circle. Use the concepts to solve problems. 6.M.8. Know and use the formulas for the volumes and surface areas of cubes and rectangular prisms, given the lengths of their sides. 6.M.9. Find the sum of the angles in simple polygons (up to eight sides) with and without measuring the angles.	7.M.1. Select, convert (within the same system of measurement), and use appropriate units of measurement or scale. 7.M.2. Demonstrate an understanding of the concepts and apply formulas and procedures for determining measures, including those of area and perimeter/circumference of parallelograms, trapezoids, and circles. Given the formulas, determine the surface area and volume of rectangular prisms and cylinders. Use technology as appropriate. 7.M.3. Demonstrate an understanding that rate is a measure of one quantity per unit value of another quantity; use models, graphs, and formulas to solve simple problems involving rates (e.g., velocity and density); check the units of the solutions; use dimensional analysis to check the reasonableness of the answer. 7.M.4. Construct and read drawings and models made to scale. 7.M.5. Use ratio and proportion, including scale factors, in the solution of problems.	8.M.1. Given the formulas, convert from one system of measurement to another. Use technology as appropriate. 8.M.2. Understand the concept of surface area and volume; given the formulas, determine the surface area and volume of rectangular prisms, cylinders, and spheres. Use technology as appropriate. 8.M.3. Use a straightedge, compass, protractor, or other tools to formulate and test conjectures and to draw geometric figures. 8.M.4. Solve problems about similar figures and scale drawings. Understand that when the lengths of all dimensions of an object are multiplied by a scale factor, the surface area is multiplied by the square of the scale factor and the volume is multiplied by the cube of the scale factor. 8.M.5. Understand and use the fact that when two polygons or circles are similar with scale factor of r, their areas are related by a factor of r^2. 8.M.6. Use proportions to express relationships between corresponding parts of similar figures.

Strand 5: Data Analysis, Statistics, and Probability

Pre-Kindergarten
PK.D.1. Graph real objects or pictures of objects (no more than three) as a way to organize information (e.g., helps to make a graph (using actual shoes) showing how many children have sneakers with Velcro and how many have laces. Places cutouts of a hamster next to his favorite name for his new classroom pet). PK.D.2. Describe and analyze information from graphs (e.g., says, "more kids like oranges than bananas," after looking at the tally marks next to the pictures of an orange and a banana. Says, "There are more boys than girls here today" after looking at the attendance graph).

Strand 5: Data Analysis, Statistics, and Probability

Kindergarten	Grade One	Grade Two
K.D.1. Gather data about self and the environment to answer questions of interest to children; record the results using concrete graphs and simple picture graphs to display data. K.D.2. Describe relationships displayed in graphs, tables, or other representations (e.g., which has the most or least number of objects?).	1.D.1. Use surveys and observations to gather data about themselves and their surroundings (e.g., what is your favorite dessert?). 1.D.2. Represent and compare data (e.g., largest, smallest, most often, least often) using tally charts, pictures, and bar graphs. 1.D.3. Ask and answer simple questions related to data representations (e.g., who is the tallest student in the class? What is the favorite fruit of the class?). 1.D.4. Decide which outcomes of experiments are certain or impossible.	2.D.1. Use interviews, surveys, and observations to gather data about themselves and their surroundings. 2.D.2. Organize, classify, and represent data using tallies, charts, tables, bar graphs, pictographs, and Venn diagrams; interpret the representations. 2.D.3. Formulate inferences (draw conclusions) and make educated guesses (conjectures) about a situation based on information gained from data. 2.D.4. Decide which outcomes of experiments are certain, impossible, or most likely. .

Strand 5: Data Analysis, Statistics, and Probability

Grade Three	Grade Four	Grade Five
3.D.1. Collect and organize data using observations, measurements, surveys, or experiments. 3.D.2. Construct, identify the main idea, and make predictions from various representations of data sets in the forms of tables, bar graphs (horizontal and vertical forms), pictographs, and tallies. 3.D.3. Record all possible outcomes for a simple event using concrete objects (e.g., tossing a coin). 3.D.4. Classify outcomes as certain, likely, unlikely, or impossible by conducting experiments using concrete objects such as counters, number cubes, spinners, or coins. 3.D.5. List and count the number of possible combinations of objects from two sets (e.g., how many different outfits can one make from a set of two sweaters and a set of three skirts?).	4.D.1. Collect and organize data using observations, measurements, surveys, or experiments and identify appropriate ways to display the data. 4.D.2. Match a representation of a data set such as lists, tables, or graphs (including circle graphs) with the actual set of data. 4.D.3. Construct, draw conclusions, and make predictions from various representations of data sets, including tables and bar graphs (where symbols or scales represent multiple units), line graphs, and line plots. 4.D.4. Compare two data sets represented in two bar graphs, pie graphs, and histograms. 4.D.5. Represent the possible outcomes for a simple probability situation (e.g., the probability of drawing a red marble from a bag containing three red marbles and four green marbles). 4.D.6. List and count the number of possible combinations of objects from three sets (e.g., how many different outfits can one make from a set of three shirts, a set of two skirts, and a set of two hats?). 4.D.7. Use the results of probability experiments to predict future events (e.g., use a line plot to predict the temperature forecast for the next day).	5.D.1. Define and apply the concepts of mean, median, and mode; compute and compare simple examples to show that they may differ. 5.D.2. Construct, label, and interpret stem-and-leaf plots, line plots, bar graphs, and circle graphs. 5.D.3. Predict the probability of outcomes of simple experiments (e.g., tossing a coin, rolling a die) and test the predictions.

Strand 5: Data Analysis, Statistics, and Probability

Grade Six	Grade Seven	Grade Eight
6.D.1. Describe and compare data sets using the concepts of median, mean, mode, maximum and minimum, and range. 6.D.2. Construct circle graphs using ratios, proportions, and percents. 6.D.3. Use tree diagrams and other models (e.g., lists and tables) to represent possible or actual outcomes of trials. 6.D.4. Represent two numerical variables on a scatterplot and describe any apparent relationship that exists between the two variables (e.g., between time spent on homework and grades in class). 6.D.5. Compute probabilities of events from simple experiments with equally likely outcomes (e.g., tossing dice, flipping coins, spinning spinners) by listing all possibilities and finding the fraction that meets given conditions. Analyze the outcomes. 6.D.6. Use appropriate ratios between 0 and 1 to represent the probability of the outcome and associate the probability with the likelihood of the event; know that 0 probability means an event will not occur and that probability 1 means an event will occur.	7.D.1. Find, describe, and interpret appropriate measures of central tendency (mean, median, and mode) and spread (range) that represent a set of data. 7.D.2. Select, create, interpret, and utilize various tabular and graphical representations of data (e.g., circle graphs, Venn diagrams, stem-and-leaf plots, histograms, tables, and charts). 7.D.3. Describe the characteristics and limitations of a data sample. Identify different ways of selecting a sample (e.g., convenience sampling, responses to a survey, random sampling). 7.D.4. Use tree diagrams, tables, organized lists, and area models to compute probabilities for simple compound events (e.g., multiple coin tosses or rolls of dice). 7.D.5. Understand that the probability of either of two disjoint events occurring is the sum of the two individual probabilities and that the probability of one event following another, in independent trials, is the product of the two probabilities.	8.D.1. Revisit measures of central tendency (mean, median, and mode) and spread (range) that represent a set of data and then observe the change in each when an "outlier" is adjoined to the data set or removed from it. Use these notions to compare different sets of data and explain how each can be useful in a different way to summarize social phenomena such as price levels, clothing sizes, and athletic performances. 8.D.2. Select, create, interpret, and utilize various tabular and graphical representations of data (e.g., scatterplots, box-and-whisker plots). 8.D.3. Recognize practices of collecting and displaying data that may bias the presentation or analysis. 8.D.4. Use data to estimate the probability of future events (e.g., batting averages). 8.D.5. Select, create, interpret, and utilize various tabular and graphical representations of data; differentiate between continuous and discrete data and ways to represent them. 8.D.6. Apply the Fundamental Counting Principle (basic combinatorics) to find total number of outcomes possible for independent and dependent events, and calculate the probabilities using organized lists or tree diagrams. 8.D.7. Understand the difference between independent and dependent events, and recognize common misconceptions involving probability (e.g., Alice rolls a 6 on a die three times in a row; she is just as likely to roll a 6 on the fourth roll as she was on any previous roll).

District of Columbia
High School Mathematics Course Standards

Algebra I

Number Sense and Operations Indicators

AI.N.1 Use the properties of operations on real numbers, including the associative, commutative, identity, and distributive properties, and use them to simplify calculations.

AI.N.2 Simplify numerical expressions, including those involving positive integer exponents or the absolute value, e.g., $3(2^4 – 1) = 45$, $4|3 – 5| + 6 = 14$; apply such simplifications in the solution of problems.

AI.N.3 Calculate and apply ratios, proportions, rates, and percentages to solve a range of consumer and practical problems.

AI.N.4 Use estimation to judge the reasonableness of results of computations and of solutions to problems involving real numbers, including approximate error in measurement and the approximate value of square roots without the use of calculators.

AI.N.5 Apply the set operations of union and intersection and the concept of complement, universal set, and disjoint sets and use them to solve problems, including those involving Venn diagrams.

AI.N.6 Demonstrate an understanding of operations with sets, including union, intersection, disjoint, and the application of Venn diagrams.

Patterns, Relations, and Algebra Indicators

AI.P.1 Describe, complete, extend, analyze, generalize, and create a wide variety of patterns, including iterative, linear, quadratic, and exponential functional relationships.

AI.P.2 Use properties of the real number system to judge the validity of equations and inequalities and to justify every step in a sequential argument.

AI.P.3 Demonstrate an understanding of relations and functions. Identify the domain, range, dependent, and independent variables of functions.

AI.P.4 Translate between different representations of functions and relations: graphs, equations, sets of ordered pairs (scatter plots), verbal, and tabular.

AI.P.5 Demonstrate an understanding of the relationship between various representations of a line. Determine a line's slope and x- and y-intercepts from its graph or from a linear equation that represents the line.

AI.P.6 Find a linear function describing a line from a graph or a geometric description of the line (e.g., by using the point-slope or slope y-intercept formulas. Explain the significance of a positive, negative, zero, or undefined slope.

AI.P.7 Find linear functions that represent lines either perpendicular or parallel to a given line and through a point (e.g., by using the point-slope form of the equation).

AI.P.8 Add, subtract, and multiply polynomials with emphasis on 1st and 2nd degree polynomials..

AI.P.9 Demonstrate facility in symbolic manipulation of polynomial and rational expressions by rearranging and collecting terms, factoring (e.g., $a^2 – b^2 = (a + b)(a - b)$, $x^2 + 10x + 21 = (x + 3)(x + 7)$, $5x^4 + 10x^3 – 5x^2 = 5x^2 (x^2 + 2x – 1)$), identifying and canceling common factors in rational expressions, and applying the properties of positive integer exponents.

AI.P.10 Divide polynomials by monomials with emphasis on 1st and 2nd degree polynomials.

AI.P.11 Find solutions to quadratic equations (with real roots) graphically, by factoring, or using the quadratic formula. Demonstrate an understanding of the equivalence of the methods. .

AI.P.12 Solve equations and inequalities including those involving absolute value of linear expressions (e.g., $|x - 2| > 5$) and apply to the solution of problems.

AI.P.13 Solve everyday problems that can be modeled using linear or quadratic functions. Apply appropriate tabular, graphical, or symbolic methods to the solution. Include compound interest, and direct and inverse variation problems. Use technology when appropriate.

AI.P.14 Solve everyday problems that can be modeled using systems of linear equations or inequalities. Apply algebraic and graphical methods to the solution. Use technology when appropriate.

AI.P.15 Use appropriate functions on the graphing calculator as tools for solving problems, testing conjectures, accessing data, and verifying solutions.

Data Analysis, Statistics, and Probability Indicators

AI.D.1 Select, create, and interpret an appropriate graphical representation (e.g., scatter plot, table, stem-and-leaf plots, circle graph, line graph, and line plot) for a set of data and use appropriate statistics (e.g., mean, median, range, and mode) to communicate information about the data. Use these notions to compare different sets of data.

AI.D.2 Approximate a line of best fit (trend line) given a set of data (e.g., scatter plot).

Geometry

G.G.1 Know correct geometric notation, including the notation for line segment (bar above *AB)* and angle (<*ABC*).

G.G.2 Recognize special types of polygons (e.g., isosceles triangles, parallelograms, and rhombuses).

G.G.3 Apply properties of sides, diagonals, and angles in special polygons; identify their parts and special segments (e.g., altitudes, midsegments); determine interior angles for regular polygons.

G.G.4 Draw and label sets of points such as line segments, rays, and circles.

G.G.5 Detect symmetries of geometric figures.

G.G.6 Identify and apply properties of basic theorems about congruent and similar figures and parallel and perpendicular lines, and use them to conjecture or deduce additional theorems, provide a proof or counterexample, and solve problems (e.g., two lines parallel to a third are parallel to each other; perpendicular bisectors of line segments are the set of all points equidistant from the two end points).

G.G.7 Distinguish between postulates and theorems. Use inductive and deductive reasoning, as well as proof by contradiction. Given a conditional statement, write its inverse, converse, and contrapositive.

G.G.8 Apply formulas for a rectangular coordinate system to justify theorems.

G.G.9 Draw congruent and similar figures using a compass, straightedge, protractor, or computer software. Make conjectures about methods of construction. Justify the conjectures by logical arguments.

G.G.10 Apply congruence and similarity correspondences (e.g., $\Delta ABC \cong \Delta XYZ$) and properties of the figures to find missing parts of geometric figures, and provide logical justification.

G.G.11 Apply properties of angles, parallel lines, arcs, radii, chords, tangents, and secants to solve problems.

G.G.12 Solve simple triangle problems using the triangle angle sum property and/or the Pythagorean theorem.

G.G.13 Use the properties of special triangles (e.g., isosceles, equilateral, 30º–60º–90º, 45º–45º–90º) to solve problems.

G.G.14 Define the sine, cosine, and tangent of an acute angle. Apply to the solution of problems.

G.G.15 Apply the triangle inequality and other inequalities associated with triangles (e.g., the longest side is opposite the greatest angle), use them to conjecture or deduce additional theorems, provide a proof or counterexample, and solve problems.

G.G.16 Demonstrate an understanding of the relationship between various representations of a line. Determine a line's slope and x- and y-intercepts from its graph or from a linear equation that represents the line. Find a linear equation describing a line from a graph or a geometric description of the line (e.g., by using the point-slope or slope y-intercept formulas). Explain the significance of a positive, negative, zero, or undefined slope.

G.G.17 Using rectangular coordinates, calculate midpoints of segments, slopes of lines and segments, and distances between two points, and apply the results to the solutions of problems.

G.G.18 Find linear equations that represent lines either perpendicular or parallel to a given line and through a point (e.g., by using the point-slope form of the equation).

G.G.19 Draw the results and interpret transformations on figures in the coordinate plane such as translations, reflections, rotations, scale factors, and the results of successive transformations. Apply transformations to the solution of problems.

G.G.20 Demonstrate the ability to visualize solid objects and recognize their projections, cross sections, and graph points in 3-D.

G.G.21 Calculate perimeter, circumference, and area of common geometric figures such as parallelograms, trapezoids, circles, and triangles.

G.G.22 Given the formula, find the lateral area, surface area, and volume of prisms, pyramids, spheres, cylinders, and cones (e.g., find the volume of a sphere with a specified surface area).

G.G.23 Relate changes in the measurement (including units) of one attribute of an object to changes in other attributes (e.g., how changing the radius or height of a cylinder affects its surface area or volume).

G.G.24 Describe the effects of approximate error in measurement and rounding on measurements and on computed values from measurements.

G.G.25 Use dimensional analysis for unit conversion and to confirm that expressions and equations make sense.

Algebra II

Number Sense and Operations Indicators

AII.N.1 Know and use the properties of operations on real numbers, including the existence of the identity and inverse elements for addition and multiplication and the existence of n^{th} roots of positive real numbers for any positive integer n, and the n^{th} power of positive real numbers.

AII.N.2 Define complex numbers (e.g., a + b*i*). Relate the system of complex numbers to the systems of real and rational numbers.

AII.N.3 Simplify numerical expressions with powers and roots, including fractional and negative exponents.

Patterns, Relations, and Algebra Indicators

AII.P.1 Describe, complete, extend, analyze, generalize, and create a wide variety of patterns, including iterative and recursive patterns such as Fibonacci Numbers and Pascal's Triangle

AII.P.2 Identify arithmetic and geometric sequences and finite arithmetic and geometric series. Use the properties of such sequences and series to solve problems, including finding the formula for the general term and the sum, recursively and explicitly.

AII.P.3 Understand functional notation, evaluate a function at a specified point in its domain, and perform operations on functions with emphasis on the domain, range.

AII.P.4 Demonstrate an understanding of the exponential and logarithmic functions.

AII.P.5 Given algebraic, numeric, and/or graphical representations, recognize functions as polynomial, rational, logarithmic, or exponential and describe their behavior.

AII.P.6 Find solutions to radical equations; find solutions to quadratic equations (with real coefficients and real or complex roots) graphically, by factoring, completing the square, or using the quadratic formula.

AII.P.7 Solve a variety of equations and inequalities using algebraic, graphical, and numerical methods, including the quadratic formula; use technology where appropriate. Include polynomial, exponential, and logarithmic functions, expressions involving the absolute values, and simple rational expressions.

AII.P.8 Explore matrices and their operations, including using them to solve systems of linear equations. Apply to solutions of everyday problems.

AII.P.9 Use symbolic, numeric, and graphical methods to solve systems of equations and/or inequalities involving algebraic, exponential, and logarithmic expressions. Also use technology where appropriate. Describe the relationships among the methods.

AII.P.10 Solve everyday problems that can be modeled using polynomial, rational, reciprocal, exponential, logarithmic, and step functions, absolute values and square roots. Apply appropriate graphical, tabular, or symbolic methods to the solution. Include compound interest, exponential growth and decay, and direct and inverse variation problems. Use technology when appropriate.

AII.P.11 Identify maximum and minimum values of functions in simple situations. Apply to the solution of problems.

AII.P.12 Recognize translations and scale changes of a given function *f*(x) resulting from substitutions for the various parameters a, b, c, and d in $y = af(b(x + c/b)) + d$. In particular, describe qualitatively the effect of such changes on polynomial, rational, exponential, and logarithmic functions.

AII.P.13 Simplify rational expressions. Solve rational equations and inequalities.

Geometry Indicators

AII.G.1 Define the sine, cosine, and tangent of an acute angle. Apply to the solution of problems.

AII.G.2 Explain the identity $\sin^2\theta + \cos^2\theta = 1$. Relate the identity to the Pythagorean theorem.

AII.G.3 Relate geometric and algebraic representations of lines and simple curves.

Data Analysis, Statistics, and Probability Indicators

AII.D.1 Select an appropriate graphical representation for a set of data and use appropriate statistics (e.g., quartile or percentile distribution) to communicate information about the data, including box plots.

AII.D.2 Use combinatorics (e.g., fundamental counting principle, permutations, and combinations) to solve problems, including computing geometric probabilities and probabilities of compound events. Use technology as appropriate.

AII.D.3 Use technology to calculate lines of best fit.

Probability and Statistics

PS.D.1 Demonstrate understanding of the definition of the notion of independent events and use the rules for addition, multiplication, and complementation to solve for probabilities of particular events in finite sample spaces.

PS.D.2 Know the definition of conditional probability and use it to solve for probabilities in finite sample spaces.

PS.D.3. Demonstrate understanding of the notion of discrete random variables by using them to solve for the probabilities of outcomes (e.g., the probability of the occurrences of five heads in 14 coin tosses).

PS.D.4. Apply uniform, normal, and binomial distributions to the solutions of problems.

PS.D.5. Determine the mean and the standard deviation of a normally distributed random variable.

PS.D.6. Know the definitions of the mean, median, and mode of a distribution of data and compute each in particular situations.

PS.D.7. Describe a set of frequency distribution data by spread (variance and standard deviation), skewness, symmetry, number of modes, or other characteristics. Use these concepts in everyday applications.

PS.D.8. Organize and describe distributions of data by using a number of different methods, including frequency tables, histograms, standard line and bar graphs, stem-and-leaf displays, scatter plots, and box-and-whisker plots.

PS.D. 9. Describe and explain how the relative sizes of a sample and the population affect the validity of predictions from a set of data.

Pre-Calculus and Trigonometry

Number Sense and Operations Indicators

PC.N.1 Define and conduct operations on complex numbers, in particular, addition, subtraction, multiplication, and division. Relate the system of complex numbers to the systems of real and rational numbers.

PC.N.2 Plot complex numbers using both rectangular and polar coordinates systems. Represent complex numbers using polar coordinates, i.e., $a + bi = r(\cos\theta + i\sin\theta)$.

PC.N.3 Apply DeMoivre's theorem to multiply, take roots, and raise complex numbers to a power.

Patterns, Relations, and Algebra Indicators

PC.P.1 Relate the number of roots of a polynomial to its degree. Solve quadratic equations with complex coefficients, including use of completing the square.

PC.P.2 Demonstrate an understanding of the trigonometric functions (sine, cosine, tangent, cosecant, secant, and cotangent). Relate the functions to their geometric definitions.

PC.P.3 Use matrices to solve systems of linear equations. Apply to the solution of everyday problems.

PC.P.4 Given algebraic, numeric, and/or graphical representations, recognize functions as polynomial, rational, logarithmic, or exponential.

PC.P.5 Combine functions by composition, as well as by addition, subtraction, multiplication, and division.

PC.P.6 Identify whether a function has an inverse and when functions are inverses of each other; explain why the graph of a function and its inverse are reflections of one another over the line y=x.

PC.P.7 Identify maximum and minimum values of functions. Apply to the solution of problems.

PC.P.8 Describe the translations and scale changes of a given function $f(x)$ resulting from substitutions for the various parameters a, b, c, and d in $y = af(b(x + c/b)) + d$. In particular, describe the effect of such changes on polynomial, rational, exponential, and logarithmic functions.

PC.P.9 Derive and apply basic trigonometric identities (e.g., $\sin^2\theta + \cos^2\theta = 1$, $\tan^2\theta + 1 = \sec^2\theta$) and the laws of sines and cosines.

PC.P.10 Demonstrate an understanding of the formulas for the sine and cosine of the sum or the difference of two angles. Relate the formulas to DeMoivre's theorem and use them to prove other trigonometric identities. Apply to the solution of problems.

PC.P.11 Understand, predict, and interpret the effects of the parameters a, ω, b, and c on the graph of $y = a\sin(\omega(x - b)) + c$; similarly for the cosine and tangent. Use to model periodic processes.

PC.P.12 Translate between geometric, algebraic, and parametric representations of curves. Apply to the solution of problems.

PC.P.13 Relate the slope of a tangent line at a specific point on a curve to the instantaneous rate of change. Explain the significance of a horizontal tangent line. Apply these concepts to the solution of problems.

PC.P.14 Approximate areas under a curve.

PC.P.15 Demonstrate an understanding of the binomial theorem and use it in the solution of problems.

Geometry Indicators

PC.G.1 Demonstrate an understanding of the laws of sines and cosines. Use the laws to solve for the unknown sides or angles in triangles. Determine the area of a triangle given the length of two adjacent sides and the measure of the included angle.

PC.G.2 Use vectors to solve problems. Describe addition of vectors, multiplication of a vector by a scalar, and the dot product of two vectors, both symbolically and geometrically. Use vector methods to obtain geometric results.

PC.G.3 Apply properties of angles, parallel lines, arcs, radii, chords, tangents, and secants to solve problems.

Measurement Indicators

PC.M.1 Describe the relationship between degree and radian measures, and use radian measure in the solution of problems, particularly problems involving angular velocity and acceleration.

PC.M.2 Use dimensional analysis for unit conversion and to confirm that expressions and equations make sense.

Data Analysis, Statistics, and Probability Indicators

PC.D.1 Design surveys and apply random sampling techniques to avoid bias in the data collection.

PC.D.2 Apply regression results and curve fitting to make predictions from data and select appropriate functions as models.

PC.D.3 Compare the results of simulations (e.g., random number tables, random functions, and area models) with predicted probabilities.

Glossary

Absolute value A number's distance from zero on the number line. The absolute value of –4 is 4; the absolute value of 4 is 4.

Addend A number that is to be added.

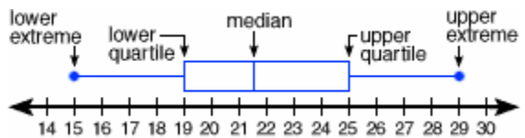
Algorithm A finite set of steps for completing a procedure, e.g., long division.

Analog Having to do with data represented by continuous variables, e.g., a clock with hour, minute, and second hands.

Associative Property Giving the same result irrespective of the order taken, thus addition and subtraction are associative, but subtraction and division are not. For addition, (a + b) + c = a + (b + c). For multiplication, (a x b) x c = a x (b x c).

Binomial Theorem A mathematical formula used to calculate the value of a two-part mathematical expression that is squared, cubed, or raised to another power or exponent without explicitly multiplying the parts themselves.

Box-and-Whisker Plot A method for displaying the median, quartiles, and extremes of a set of data using the number line.



Calculus The mathematics of change and motion. The main concepts of calculus are limits, instantaneous rates of change, and areas enclosed by curves.

Capacity The maximum amount or number that can be contained or accommodated, e.g., a jug with a one-gallon capacity; the auditorium was filled to capacity.

Cartesian Coordinate Plane A coordinate plane with perpendicular coordinate axes.

Closure Property A set of numbers, such as the integers, is closed under a particular operation if performing the operation on numbers in the set results in another number in that set. For example, the set of non-zero integers is closed under multiplication but is not closed under division the product of two non-zero integers is again a non-zero integer, but their quotient need not be an integer.

Commutative Properties Giving the same result in mathematics or logic irrespective of the order in which two or more terms or quantities are placed. For example, a + b = b + a.

Complex Number A number that can be written in the form a + bi where a and b are real numbers and i = $\sqrt{-1}$.

Composite Numbers A natural number that is not prime.

Congruent Two shapes in the plane or in space are congruent if there is a rigid motion that identifies one with the other.

Convenience (Haphazard) Sampling The collection of data from readily available sources without emphasis on methodological rigor.

Coordinate Plane A plane in which two coordinate axes are specified, i.e., two intersecting directed straight lines, usually perpendicular to each other, and usually called the x-axis and y-axis. Every point in a coordinate plane can be described uniquely by an ordered pair of numbers, the coordinates of the point with respect to the coordinate axes.

Decimal Number Any real number expressed in base 10 notation, such as 2.673.

Deductive Reasoning A type of reasoning wherein the conclusion about particulars follows necessarily from general or universal premises.

Digit Any of the Arabic numerals 1 to 9 and usually the symbol 0; one of the elements that combine to form numbers in a system other than the decimal system.

Digital Having to do with data that is represented in the form of numerical digits; providing a readout in numerical digits, e.g., a digital watch.

Dilation A type of transformation of the plane that fixes a point C (the center of the dilation) and maps any other point P to the point P' characterized as follows: the segment CP' has the same direction as the segment CP, and has length k times the length of the segment CP where k is a positive constant (the scale factor of the dilation).

Divisibility Rules Patterns that allow you to determine whether or not a number divides evenly into another number (leaving no remainder) without actually doing the division, e.g., a number that ends in 5 or 0 is divisible by 5.

Dot Product For vectors A = <xa, ya> and B = <xb, yb>, the dot product A•B = (xa)(xb) + (ya) (yb).

Equilateral Used to describe a geometric figure in which all sides are of equal length.

Expanded Notation A way of representing a number that shows the sum of each digit multiplied by the appropriate positive power of ten and the units digit, e.g., 3451 as 3 x 1000 + 4 x 100 + 5 x 10 + 1 or as 3 x 103+ 4 x 102 + 5 x 10 + 1.

Exponent The number that indicates how many times the base is used as a factor, e.g., in 43 = 4 x 4 x 4 = 64, the exponent is 3, indicating that 4 is repeated as a factor three times.

Fact Families A fact family is a set of math problems that use the same three numbers. For example, the numbers 1, 13 and 14 can be used to make two addition problems, 1 + 13 = 14 and 13 + 1 = 14, and two subtraction problems, 14 – 13 = 1 and 14 – 1 = 13.

Factors Any of the two or more quantities that are multiplied together. In the expression 3.712 X 11.315, the factors are 3.712 and 11.315.

Fibonacci Numbers The sequence of numbers beginning with 1, in which each number that follows is the sum of the previous two numbers, i.e., 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144….

Function A mathematical relation that associates each object in a set with exactly one value.

Fundamental Counting Principle If event M can occur in m ways and, after it has occurred, event N can occur in n ways, then event M followed by event N can occur m•n ways.

Geometric Pattern A sequence of symbols or geometric figures.

Geometric Sequence (Progression) An ordered list of numbers that has a common ratio between consecutive terms, e.g., 2, 6, 18, 54….

Greatest Common Factor The greatest common factor of two numbers, a and b, is the largest number that divides both a and b evenly.

Histogram A vertical block graph with no spaces between the blocks. It is used to represent frequency data in statistics.

Identity Properties For addition, a + 0 = a. For multiplication, a x 1 = a.

Inductive Reasoning a) The type of reasoning that uses inference to reach a generalized conclusion from particular instances; b) In mathematics, demonstration of the validity of a law concerning all the positive integers by proving that it holds for the integer 1 and that if it holds for an arbitrarily chosen positive integer k it must hold for the integer k+1; also called mathematical induction.

Integer A number that is either a whole number or the negative of a whole number.

Irrational Number A number that cannot be expressed as a quotient of two integers, e.g., $\sqrt{2}$. It can be shown that a number is irrational if and only if it cannot be written as a repeating or terminating decimal.

Isosceles Triangle Used to describe a triangle in which two of the three sides are of equal length.

Iterative Pattern or Sequence A sequence or pattern formed by repeating the same procedure, for example, the Fibonacci sequence.

Least Common Multiple The least common multiple of two numbers, a and b, is the smallest number that contains both a and b as factors.

Line Graph A set of data points on an X-Y grid, possibly with consecutive points connected by line segments.

Line of Best Fit A straight line drawn through or near to as many data points as possible on a scatter plot.

Line Plot A number line with dots or other marks above it to show the number of times an event occurs.

Linear Equation Any equation that can be written in the form Ax + By + C = 0 where A and B cannot both be 0. The graph of such an equation is a line.

Manipulatives Various objects or tools used to teach abstract concepts through hands-on activities.

Matrix (pl. Matrices) A rectangular array of numbers or variables.

Mean In statistics, the average obtained by dividing the sum of two or more quantities by the number of these quantities.

Measure of Central Tendency Either the mean, median, or mode of a distribution, i.e., one of the possible notions of an average.

Median In statistics, the quantity designating the middle value in a set of numbers.

Mixed Number A number that is written as a whole number and a fraction.

Mode In statistics, the value that occurs most frequently in a given series of numbers.

Monomial In the variables x, y, z, a monomial is an expression of the form axmynzk in which m, n, and k are nonnegative integers and is a constant (e.g., 5x2, 3x2 y or 7x3yz2).

Nets An arrangement of two-dimensional figures that can be folded to make a polyhedron (a solid figure with flat faces that are polygons).

Nonstandard Unit Unit of measurement expressed in term of objects (such as paper clips, sticks of gum, shoes, etc.).

Numeral A symbol or mark used to represent a number.

Numeric Pattern A pattern composed of numerals.

Order of Operations 1. Do all of the operations inside parentheses and/or above and below a fraction bar in the proper order; 2. Find the value of any powers or roots; 3. Multiply and divide from left to right; 4. Add and subtract from left to right.

Ordinal Number A number designating the place (as first, second, or third) occupied by an item in an ordered sequence.

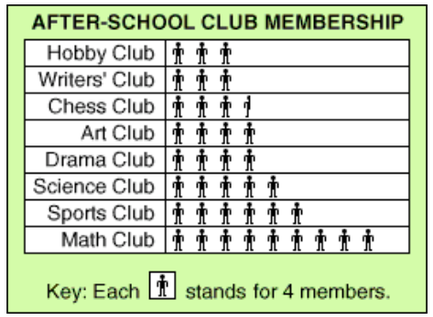
Parallel Given distinct lines in a plane that are infinite in both directions, the lines are parallel if they never meet. The distinct lines in the coordinate plane are parallel if and only if they have the same slope.

Partitioning A process of dividing an object into parts.

Pascal's Triangle A triangular arrangement of numbers in which each row starts and ends with 1 and each other number is the sum of the two numbers above it.

Perpendicular At right angles to a line or plane.

Pictogram (Pictograph) A graph that uses pictures to show and compare information.



Polynomial In algebra, a sum of monomials; for example, x 2+ 2xy + y2.

Prime Factorization A number written as the product of all its prime factors.

Prime Number A whole number that can only be divided without a remainder by itself and one. The first seven primes are 2, 3, 5, 7, 11, 13, 17.

Probability A number between zero and one that describes the likelihood that a given event will take place. For example, the probability of throwing a six with a single throw of one die is 1/6 and the probability of throwing two sixes with a single throw of two dice is 1/36.

Proof A method of constructing a valid argument using deductive reasoning.

Proportion An equation that states that two ratios are equivalent, e.g., 4/8 = 1/2 or 4 : 8 = 1 : 2.

Pythagorean Theorem For any right triangle, the sum of the squares of the measures of the legs equals the square of the measure of the hypotenuse.

Quadratic Function A function given a polynomial of degree 2.

Quadrilateral A two-dimensional geometric figure with four sides.

Random Sampling A smaller group of people or objects chosen from a larger group or population by a process giving equal chance of selection to all possible people or objects.

Random Variable A variable that is itself a function of the result of a statistical experiment in which each outcome has a definite probability of occurrence; also called variate.

Range In statistics, the difference between the smallest and the largest values in a frequency distribution.

Ratio A comparison of two numbers or quantities, e.g., 4 to 7 or 4 : 7 or 4/7.

Rational Number A number that can be written as the ratio of an integer to a counting number; or more formally, a number that can be expressed as a ratio a/b where a and b are integers and b ≠ 0, e.g., 0.5, 3/5, -3, 8, 3, 9/10.

Real Number A number from the set of numbers consisting of all rational and all irrational numbers.

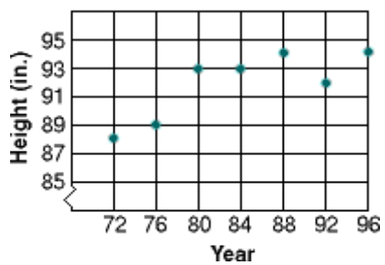
Recursive Pattern or Sequence A pattern or sequence wherein each successive term can be computed from some or all of the preceding terms by an algorithmic procedure.

Reflection A type of transformation that flips points about a line, called the *line of reflection*. Taken together, the image and the pre-image have the line of reflection as a line of symmetry.

Rotation A type of transformation that turns a figure about a fixed point, called the *center of rotation*.

Sample Space In probability, the set of all outcomes of a given experiment, e.g., the sample space for tossing two coins is (H,H), (H,T), (T,H), (T,T).

Scatter Plot Two sets of data plotted as ordered pairs in the coordinate plane.



Scientific Notation A widely used floating-point system in which numbers are expressed as products consisting of a number between 1 and 10 multiplied by an appropriate power of 10, e.g., 562 = 5.62 x 102.

Sequence A set of elements ordered so that they can be labeled with consecutive positive integers starting with 1, e.g., 1, 3, 9, 27, 81. In this sequence, 1 is the first term, 3 is the second term, 9 is the third term, and so on.

Similarity In geometry, two shapes R and S are similar if there is a dilation D (see the definition of dilation) that takes S to a shape congruent to R. It follows that R and S are similar if they are congruent after one of them is expanded or shrunk.

Square Root A number or quantity that when multiplied by itself gives the stated number or quantity. The square roots of 16 are 4 and –4. The square roots of –16 are 4i and –4i.

Symmetry A symmetry of a shape S in the plane or space is a rigid motion T that takes S onto itself (T(S) = S). For example, reflection through a diagonal and a rotation through a right angle about the center are both symmetries of the square.

Transformation A prescription, or rule, that sets up a one-to-one correspondence between the points in a geometric object (the pre-image) and the points in another geometric object (the image). Reflections, rotations, translations, and dilations are particular examples of transformations.

Translation A type of transformation that moves every point by the same distance in the same direction, e.g., on a geographic map, moving a given distance due north.

Tree Diagram A diagram of a hierarchical structure that shows the relationships between components as branches.

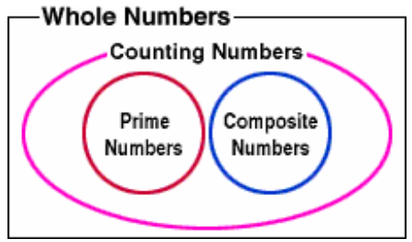
Turn Around Facts The commutative property of addition (e.g., 2 + 5 = 7, 5 + 2 = 7).

Valid Well-grounded or justifiable; being at once relevant and meaningful (e.g., a valid theory); logically correct.

Variable A letter used to represent one or more numbers in an expression, equation, inequality, or matrix.

Vector A quantity that has magnitude and direction. A vector is typically represented by a directed line segment whose length represents the magnitude and whose orientation in space represents the direction.

Venn Diagram A diagram that is used to show relationships between sets.



Whole Number A number that is either a counting number or zero.

District of Columbia Public Schools
Office of the Chief Academic Officer
825 North Capitol Street, NE, Eighth Floor
Washington, DC 20002-4232
(202) 442-5599
Fax (202) 442-5602
www.k12.dc.us

Sue P. White
Director of Mathematics
Email: Sue.White@k12.dc.us